Discoveries Of India

Aryabhatta (476 CE) approximated Π = 62832/20000 = 3.1416. Correct to 4 decimal places, much better than the 22/7 taught in all of our schools even now

The last couple of months have seen a lot of sound and fury about the achievements of the Indic civilisation. Some have claimed that India invented spaceships; others, that our rishis knew about the spin of an electron. Swarajya cuts through all the hype and hullabaloo to take a look, based on pure hard evidence, at what ancient Indians knew, discovered and invented. There is much that we should be proud of, there is much that we shoud celebrate. There is much to teach our children.

The Story Of Our Numbers

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THE HISTORY OF mathematics is a vast topic which can never be studied perfectly since much of the work of ancient times remains undiscovered or has been lost through time. However, there is also much that is known and many important discoveries have been made over the past 150 years that debunk the theory that mathematics is a European "invention". The truth is being restored in academic circles about the historical development of mathematics and India's contribution to it.

Once the dots are connected, it is not difficult to imagine the continuation of the same pursuit of knowledge and excellence of solutions by Indus, Vedic and post-Vedic Indian minds. When seen in this thread, it seems all but natural to see the advancements of the Indus Valley civilization (standardized weights in binary sequence, the world's first measuring ruler, proto-dentistry, advanced metallurgy) to Baudhayana in 800 BCE (the so-called Pythagoras theorem), Pingala in 300 BCE (combinatorics, the mathematics of finite or countable dicrete structures), Aryabhatta in 499 CE (trigonometry) and Madhava in 1400 CE (calculus) with many in between.

While some will always turn a blind eye to facts staring in the face, there are many who would like to know the reality but can't access or comprehend the greatness of the work. I am not adding any original research of mine to the history of math, and have drawn a lot from the wonderful and pivotal book *The History of Ancient Indian Mathematics* by C.N. Srinivas Iyengar. My humble contribution to this discussion is merely to present the relevant facts in a simple way and connect the dots.

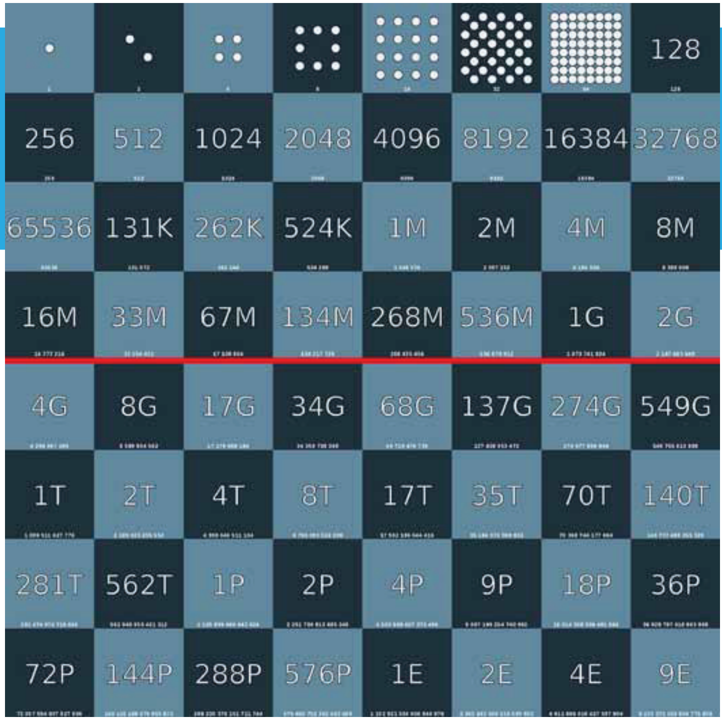
Labeling Science

The labels used in the study of history of science or math are Egyptian, Greek, Mesopotamian, Indian, European, Hindu, Islamic or Christian.

The Tower of Brahma

This is the ancient legend. In the Kashi Vishwanath temple, there are three time-worn posts in a room with 64 golden discs of reducing sizes, stacked on top of each other on one post, smallest on top. The entire stack of golden discs has to be moved to the third post, obeying the following simple rules:
Move only one disc at a time.
Only the uppermost disc can be moved.
A disc can't be on top of a smaller one.
The second post can be used for intermediate moves.

Priests have been moving these discs, in accordance with the immutable rules of Brahmā the Creator since the beginning of time. But here is the doomsday prophecy: "When the last move of the puzzle will be completed, the world will be recycled." With 3 discs it takes 7 moves, with 10 it takes 1,023 moves, for n discs it takes 2^n -1 moves. If the priests were to move discs at a rate of one per second, never making a mistake, it would take them $2^{64}-1$ seconds, or roughly 585 billion years or 18,446,744,073,709,551,615 turns to finish, or about 127 times the current age of the sun. What is amazing is that this large an astronomical number is given as the age of the Creation. The current accepted age of the Big Bang is about 13 billion years. With new developments in astronomy, this estimate may also get updated, as time is now also being thought of as not just relative but even cyclic. A puzzle based on this legend was invented by the French mathematician Edouard Lucas in 1883. Known as Tower of Hanoi puzzle, it's a very common problem in Computer Science courses to teach recursion.



I think it is unfair to use any religion's name with scientific development. None of the book-based religions fostered or encouraged science; in fact, they hindered and quashed science as is evident from the history of Europe.

While it is okay to refer to Greek or European math and Arabic or Indian math, it is not right to say Islamic or Christian math.

Only in India do you find that the pursuit of faith and the pursuit of fact went side by side amicably.

Giving Credit Where It Is Due

One might also wonder if it makes any difference to world hunger and global warming if this record is set straight. While academic researchers know the true history, does it matter if our school and college students know it as well? After all, these are trivial things like basic trigonometry or Pythagoras theorem or zero. Math has come such a long way ahead, who cares if it was Pythagoras or Baudhayana?

But, if it really doesn't matter, then why do we even call it Pythagoras theorem, Euclid's geometry or Newton's power series for sine? We should simply call them by their function, like the Hypotenuse theorem, geometry, or Power Series for sine.

Why should we take the trouble to remember the people who discover or do something for the first time, in science or in life?

Even in divine matters, we keep comparing who said what and when, and who said it first! Maybe we do so to give credit where it is due? And human achievements inspire us?

Why should India not have its inspiration? And have its credit where it is due? Everyone with a basic education in sciences should know about the great Indian minds. Why should some Indian names not become household names, at least in India, through the education system? The first step towards that is to understand, realize and appreciate how our ancestors pursued knowledge.

Indian scholars made vast contributions to mathematical astronomy and thus contributed mightily to the development of arithmetic, algebra, trigonometry and secondarily geometry (although this topic was well developed by the Greeks) and combinatorics.

Perhaps most remarkable were the developments in the fields of infinite series expansions of trigonometric expressions and differential calculus.

Surpassing all these achievements however was the development of decimal numeration and the place value system, which, without doubt, stand together as the most remarkable developments in the history of mathematics. The decimal place value system allowed the

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subject of mathematics to be developed in ways that, to put it simply, would not have been possible otherwise.

We would still be a humanind as depicted (so frightfully entertainingly) in Asterix comics.

The Eurocentric Worldview

The standard of evidence required to claim transmission of knowledge from East to West is greater than the standard of evidence required for knowledge traveling from West to East.

In the Eurocentric view, the ancient Greeks are the epitome of logic, scientific temperament, and mathematical achievement, whereas Indians "indulge in flights of fancy with airplanes in Vedic times or *rishis* travelling in space". One of the most talked about recent examples in the media is the Pythagoras theorem and India's claim to it. This is also the simplest one to understand.

PYTHAGORAS

Most of the information about Pythagoras (570-495 BCE) was written down centuries after he passed away, so very little reliable information is known about him. No texts by Pythagoras are known to have survived.

Diogenes Laërtius reported that Pythagoras had undertaken extensive travels, having not only visited Egypt but also "journeyed among the Chaldaeans and Magi", for the purpose of collecting all available knowledge and especially looking for information concerning the secret or mystic cults of the gods. Around 530 BCE, he moved to Croton, in Magna Graecia, and set up a religious sect. His followers pursued the rites and practices developed by Pythagoras and studied his philosophical theories. He was more known for his philosophy than his mathematics in his own times.

In time, Pythagoras became the subject of elaborate legends. Aristotle described Pythagoras as a wonder-worker and somewhat of a supernatural figure, attributing to him such aspects as a golden thigh, which was a sign of divinity. Some ancients believed that he had the ability to travel through space and time and to communicate with animals and plants. Another legend claims that Pythagoras asserted he could write on the moon.

EUCLID

The long-lasting nature of *The Elements* must make Euclid (4th and 3rd century BCE) the leading mathematics teacher of all time. However, little is known of Euclid's life except that he taught at Alexandria in Egypt. He is rarely mentioned by name by other Greek mathematicians from Archimedes onward. The few historical references to Euclid were written centuries after he lived, by Proclus, 450 CE and Pappus of Alexandria, 320 CE.

A detailed biography of Euclid is by Arabian authors, mentioning, for example, a birth town of Tyre. But this story of Euclid's life is generally believed to be completely fictitious. Because the lack of biographical information is unusual for the period, some researchers have proposed that Euclid was not, in fact, a historical character and that his works were written by a team of mathematicians.

Proclus, the last major Greek philosopher, who lived around 450 CE wrote: "Euclid, who put together *The Elements*, arranging in order many of Eudoxus's theorems, perfecting many of Theaetetus', and also bringing to irrefutable demonstration the things which had been only loosely proved by his predecessors. This man lived in the time of the first Ptolemy (of Egypt)."

Probably no results in *The Elements* were first proved by Euclid but the organisation of the material and its exposition are certainly to his credit.

Connecting The Dots

How can we say that Indian mathematicians actually had an impact, that too a transforming impact, on the world stage? Let us simply look at some verifiable facts from history (turn the page and look at the chart Mathematical Milestones), and you can connect the dots. Most of this information is also easily available on the net as well.

But there is an important question that needs to be asked.

Why is it that there was no advance in math in Europe from 230 BCE to 1572 CE, a full 18 centuries? And why is it that all of a sudden from 17th century onwards, we see hundreds of new mathematicians in Europe?

Here is what was happening in another part

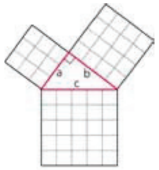
How Many Rice Grains?

Look to your left. A chessboard with numbers rather than the pieces. Chess originated in India during the Gupta Empire (4th to 6th century CE). Its early form in the 6th century was known as *chaturanga* (and later as *shataranja* in Persian), meaning "four divisions (of the military)"—infantry, cavalry, elephantry, and chariotry. These forms are represented by the pieces that would evolve into the modern pawn, knight, rook and bishop respectively. *Chaturanga* was famously played in the *Mahabharata*. Findings in Mohenjo-daro and Harappa (2600–1500 BCE) include a board game that resembles chess.

An interesting story goes that a king who was defeated by an opponent (some say it was to the craftsman who built a beautiful board specially for the king) told the man to ask his prize. What he humbly asked was just a few grains of rice: "On the first square just a single rice grain, two on the second, and similarly doubling the amount on subsequent squares, O King." The king felt insulted. But 1 + 2 + 4 + 8 + ... + 263, the total amount on the entire chessboard would be $2^{64} - 1$ = 18,446,744,073,709,551,615 grains of rice, weighing 461 billion tonnes, which would be a heap of rice larger than Mount Everest. This is around 1,000 times the global production of rice in 2010 (464 million tonnes). That would be allegedly sufficient to cover the whole territory of India with a meter thick layer of rice. At 10 grains of rice per square inch, the above amount requires rice fields covering twice the surface area of the Earth, oceans included. Incidentally, the number of grains is same as the number of moves of the Tower of Brahma problem!

Mathematical Milestones

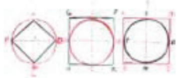
Prior to **850 BCE**, we have evidence of mainly Mesopotamian and Egyptian mathematics in the western hemisphere. Indus Valley Civilization shows remarkable advances in math, science and technology.



800 BCE
Baudhayana Sulva Sutra (and six other *sutras*) states and uses the 'Pythagoras' theorem, and calculates the square root of 2 accurately till 5 decimal places. It covers the books 1, 2, and 6 of Euclid's *Elements* to come in 300 BCE, 500 years later.

575 BCE
Thales brings Babylonian knowledge to Greece. Greek awakening starts.

300 BCE
Bhadrabahu postulates five types of infinity—ray, line, area, volume, and perpetual. Deals with numbers as large as 10^{207}. Euclid's *Elements*.



2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97...

230 BCE
Eratosthenes of Cyrene develops his sieve method for finding all prime numbers.

No major new work comes out of Greece/Europe except commentaries and compilations.

Dark Ages of Europe coincides with this period, with conflict of religion and science.

1202
Fibonacci writes *Liber Abaci* (*Book of Abacus*) and updated in 1254, sets out arithmetic and algebra he learned in Arabia and Persia. He introduces the Hindu number system using 9 digits and a symbol for zero, the decimal place value system, and makes a case for this new way of doing math. He gets no support. He produced the first significant mathematics in Europe since the time of Eratosthenes (230 BCE).

1572
Bombelli gives first new contribution. He is the first to state the rules for calculating with complex numbers. All others before him wrote what was already discovered elsewhere.

1614
Napier publishes his work on logarithms.

Magic Square

The 3×3 magic square has been a part of rituals in India since Vedic times, and still is today. The Ganesh *yantra* is a 3×3 magic square. There is a well-known 10th-century 4×4 magic square on display in the Parshvanath Jain temple in Khajuraho, India.

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

This is known as the *Chautisa* (meaning 34) Yantra. Each row, column, and diagonal, as well as each 2×2 sub-square, the corners of each 3×3 and 4×4 square, the offset diagonals and the sum of the middle two entries of the two outer columns and rows, sums to 34. Figure it out!

of the world—India and Arabia—during this period between Eratosthenes and Bombelli. Only major achievements are mentioned, a lot more happened between the lines.

1st century BCE (some date it to 1st century CE), *Lalita Vistara* mentions the Buddha enumerating to a mathematician Arjuna the multiples of 100 starting from 107 to 1,053. Taking this as a first level, he then carries on and gets eventually to 10,421 (the *Ramayana*, a much earlier work, has names for number as high as 1,055 in *Yuddha Kanda*, *sarga* 28, where Ravana's spy reports about the strength of Rama's army)

499 Aryabhatta of Kusumpura (modern Patna) composes *Aryabhatya*, gives best value of pi to 5 decimal places, trigonometric tables accurate to 0.03 per cent, gives formulae for calculating sine for intermediate angles, develops word numerals, *kuttaka* method of solving indeterminate equations; solves cube roots.

628 Brahmagupta composes *Brahmasphuta Siddhanta*. Uses zero, negative numbers, solves so-called Pell's (1668) equation $Nx^2+1=y^2$, geometric progressions, formula for finding "Pythagorean" triplets and methods for calculating sine of intermediate angles from the sine tables.

762 Baghdad city founded by Abu Jafar al-Mansur with its famed "House of Wisdom" centre of learning.

770 Kanka, a scholar from Ujjain, invited to Baghdad to explain Hindu arithmetic and astronomy. *Brahmasphuta Siddhanta* translated by Al Fazari into Arabic and named *Sind Hind* (for *siddhanta*). Another translation of Indian astronomy, *Alarkand*, may have been that of *Surya Siddhanta*.

820 al-Khwarizmi writes *The Book of Addition and Subtraction According to the Hindu Calculation* (*Kitab al-Jam wa-l-tafriq bi-hisab al-Hind*). The original Arabic does not survive.

About 830 al-Khwarizmi writes *The Compendious Book on Calculation by Completion and Balancing* (*al-Kitab al-mukhtasar fi hisab al-jabr wa'l-muqabala*). From the word *al-jabr*, we get the word algebra.

850 Mahavira writes *Ganita Sara Sangraha*, works on unit fractions, combinatorics and fractions.

900 Sridhara is first mathematician to give a rule to solve a quadratic equation.

About 1120 Adelard of Bath translates Sridhara's book in Latin, which survived.

1150 Bhaskara writes *Siddhanta Shiromani* with works on arithmetic, algebra, spheres and astronomy; advances operations on zero and concept of infinity; advances knowledge on permutations and combinations; first sure signs of differential calculus and Rolle's theorem (1691) when finding instantaneous speed of a planet; leaps in trigonometric formulae; refines kuttaka method; solves so-called Pell's equation using his *chakravala* method;

12th century European scholars travel to Spain and Sicily seeking scientific Arabic texts, including al-Khwarizmi's *The Compendious Book on Calculation by Completion and Balancing*, translated into Latin by Robert of Chester.

1202 Fibonacci publishes *Liber Abaci* and introduces the Indian number system to Europe. He finds no takers.

About 1400 Madhava of the Kerala school of mathematicians gives what later will be called Gregory series (1667), Newton power series (1665), Maclaurin series (1740), Leibniz power series for pi (1673), Euler series (1727), Taylor series (1715), all important results of calculus and series.

The first person in modern times to realize that the mathematicians of Kerala had anticipated some of the results of the Europeans on the calculus by nearly 300 years was Charles Whish in 1835. But Whish's paper, published in the *Transactions of the Royal Asiatic Society of Great Britain and Ireland*, went unnoticed by historians of mathematics. Only 100 years later, in the 1940s, did the world look in detail at the works of Kerala's mathematicians and find that the remarkable claims made by Whish were essentially true.

As Dr George Gheverghese Joseph of the University of Manchester puts it:

"Europe had a 500-year-old tradition of importing knowledge and books from India and the Arab world. There was plenty of opportunity to collect the information as European Jesuits were present in Kerala at that time. They were learned with a strong background in math and were well versed in the local languages. And there was strong motivation: Pope Gregory XIII set up a committee to look into modernizing the Julian calendar.

"On the committee was the German Jesuit astronomer/mathematician Clavius who repeatedly requested information on how people constructed calendars in other parts of the world. The Kerala School was undoubtedly a leading light in this area. Large prizes were offered to mathematicians who specialized in astronomy. Again, there were many such requests for information across the world from leading Jesuit researchers in Europe. Kerala mathematicians were hugely skilled in this area."

Ian Pearce of St Andrews University, Scotland, explains (emphasis mine):

"The Greeks however did not adopt a positional number system. I*t is worth thinking just how significant this fact is. The Greek mathematical achievements were based on geometry. Although Euclid's* Elements *contains a book on number theory, it is based on geometry. In other words, Greek mathematicians did not need to name their numbers since they worked with numbers as lengths of lines.*

"The [Indian] system also spread to Spain in the 12th century. It took much longer for the system to be accepted in mainland Europe, but eventually, by the 16th century, it was widely used. *That said, both prejudice and suspicion continued to be widespread, while orthodoxy also played its part in the continued use of Roman numerals.*

"*Cardan solved cubic and quartic equations without using zero. He would have found his work in the 1500s so much easier if he had had a zero but it was not part of his mathematics.*"

What should one do after knowing all this? This is already part of school textbooks, and elementary for today's math.

The best thing modern India can do with this knowledge of its history is perhaps this. Understand the original manuscripts with the help of math scholars and Sanskrit pundits to see if there are any insights to new ways of looking at things available.

Get a sense that the Indian thought system and society not only produced great minds but also accepted the pursuit of truth without any conflict with or oppression by "religious" thinkers. Take inspiration from the glorious pursuits of our forefathers, just like you would from Newton, Gauss, Feynman or Einstein, but much closer to home. And students can also take some hope and pride to pursue pure sciences as well, rather than just go to IT in droves.

Awaken. Arise. Advance.

You can do it. They already did.

Parallel Universe, Anyone?

"And who will search through the wide infinities of space to count the universes side by side, each containing its Brahma, its Vishnu and its Shiva? Who can count the Indra-s in them all—those Indras side by side, who reign at once in all the innumerable worlds; those others who passed away before them; or even the Indra-s who succeed each other in any given line, ascending to godly kingship, one by one, and, one by one, passing away?" (*Brahma Vaivarta Purana*)

American theoretical physicist and string theorist Brian Greene discusses nine types of parallel universes in his book *The Hidden Reality: Parallel Universes and the Deep Laws of the Cosmos*:

The quilted multiverse works only in an infinite universe. With an infinite amount of space, every possible event will occur an infinite number of times. However, the speed of light prevents us from being aware of these other identical areas.

The inflationary multiverse is composed of various pockets where inflation fields collapse and form new universes.

The brane multiverse follows from M-theory and states that each universe is a 3-dimensional brane that exists with many others. Particles are bound to their branes except for gravity.

The cyclic multiverse has multiple branes (each a universe) that collided, causing Big Bangs. The universes bounce back and pass through time, until they are pulled back together and again collide, destroying the old contents and creating them anew.

The landscape multiverse relies on string theory's Calabi–Yau shapes. Quantum fluctuations drop the shapes to a lower energy level, creating a pocket with a different set of laws from the surrounding space.

The quantum multiverse creates a new universe when a diversion in events occurs, as in the many-worlds interpretation of quantum mechanics.

Holographic multiverse: the theory is that that surface area of a space simulate the volume of the region.

The simulated multiverse exists on complex computer systems that simulate entire universes.

The ultimate multiverse contains every mathematically possible universe under different laws of physics.